

CUMULATIVE DISTRIBUTION TO THE EMPIRICAL CONTINUOUS DISTRIBUTION FUNCTION 2

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Abstract: This paper proposes the new cumulative distributive function for density function

$$f(x) = \begin{cases} 2\lambda'P(n, x) - P(n-1, x) & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{for the chosen random variable "how likely there$$

are successive departures in a particular interval".

Key Words: Random variable, Continuous probability distribution, Departure rate, Density function, Cumulative Distribution Function.

Introduction Statistics has the most interesting solutions for the problems in several fields due to its universality. Several new distributions have been developed by taking some subtle transformations on the existing distributions. This paper is continuous work to the previous one [2] which is briefed here. The Random variable of interest is to "how likely there are successive departures in a particular interval". Instead of asking "how many departures take place in a particular time interval (Poisson)", we ask for "how likely there are successive departures in a particular interval". Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is the distributions of X . We charted the histograms for successive arrivals in a particular interval from which we found the density curves.

In which case its probability density function is given by

$$f(x) = \begin{cases} 2\lambda'P(n, x) - P(n-1, x) & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $P(n, t)$ is the Truncated Poisson probability distribution of remaining n customers in the queuing system after departing $N-n$ customers from the queuing system in time interval $[0, t]$

$P(n-1, t)$ is the Truncated Poisson probability distribution of remaining $n-1$ customers in the queuing system after departing $N-(n-1)$ customers from the system in time interval $[0, t]$ and \mathfrak{S}' is the normalizing constant.

Graphs of density function:

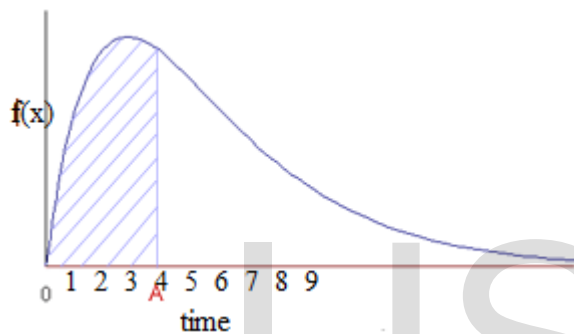


Figure 1

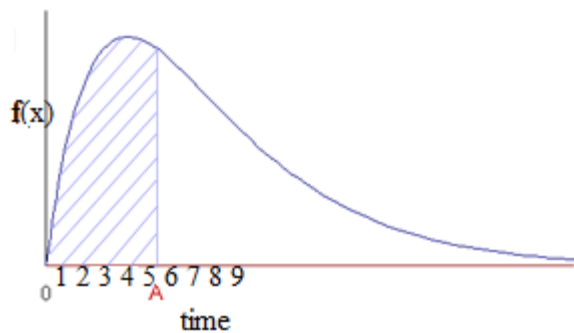


Figure 2

To extend this distribution theory, the cumulative distributive function and interesting properties of all statistical distributions mean, variance and standard deviation are studied which are widely used in several fields insurance, management, business and finance etc.

Cumulative distributive function (CDF) $F_X(x)$

It is the probability that the random variable takes values less than or equal to certain number.

$$F_X(x) = P(x \leq x) = \int_{-\infty}^x f_X(t) dt$$

And derivative of CDF is equal to the density

$$\frac{d}{dx}(F_X(x)) = f_X(x)$$

Cumulative Distribution Function of the above density function is

$$F_X(x) = \begin{cases} 1 - \int_x^{\infty} f(t) dt & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - \int_x^{\infty} \mu [2\mathfrak{Z}'P(n,t) - P(n-1,t)] dt & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $P(n, t)$ is the Truncated Poisson probability distribution of remaining n customers in the queuing system after departing $N-n$ customers from the queuing system in time interval $[0, t]$

$P(n-1, t)$ is the Truncated Poisson probability distribution of remaining $n-1$ customers in the queuing system after departing $N-(n-1)$ customers from the system in time interval $[0, t]$ and \mathfrak{Z}' is the normalizing constant.

CONCLUSION

In this paper proposed the Cumulative Distributive Function for density function

$$f(x) = \begin{cases} 2\mathfrak{Z}'P(n,x) - P(n-1,x) & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{for the chosen random variable "how likely there$$

are successive departures in a particular interval".

$$F_x(x) = \begin{cases} 1 - \int_x^\infty \mu [2\mathfrak{I}'P(n,t) - P(n-1,t)] dt & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{where } P(n, t) \text{ is the Truncated}$$

Poisson probability distribution of remaining n customers in the queuing system after departing $N-n$ customers from the queuing system in time interval $[0, t]$

$P(n-1, t)$ is the Truncated Poisson probability distribution of remaining $n-1$ customers in the queuing system after departing $N-(n-1)$ customers from the system in time interval $[0, t]$ and \mathfrak{I}' is the normalizing constant.

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